ON THE MODULARITY OF CALABI–YAU VARIETIES

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Abstract

A smooth projective variety X is called a *Calabi–Yau* variety if
(1) $H^i(X, \mathcal{O}_X) = 0$ for every $i, 0 \leq i \leq dim(X)$, and (2) the canonical bundle K_X is trivial.

Elliptic curves are Calabi–Yau varieties of dimension one, and K3 surfaces are dimension two Calabi–Yau varieties. Dimension three ones are Calabi–Yau threefolds.

In this lecture, I will address the modularity questions (conjectures) for Calabi–Yau varieties defined over $\mathbf{Q}$.

The modularity has been established for all elliptic curves over $\mathbf{Q}$ by the celebrated effort of Wiles et al. For dimension two case, the modularity conjecture has been proved for *extremal* K3 surfaces over $\mathbf{Q}$ by R. Livné (though the modularity question is still open for non-extremal K3 surfaces).

Therefore, my lecture will be concentrated on the modularity questions for Calabi–Yau threefolds defined over $\mathbf{Q}$. Calabi–Yau threefolds are classified into two distinct classes, that is, *rigid* Calabi–Yau threefolds (for which the mirror symmetry conjecture fails), and *non-rigid* Calabi–Yau threefolds. The modularity conjecture can be formulated for rigid Calabi–Yau threefolds as a special case of the Fontaine–Mazur conjecture. Up to date, there are about 45 rigid Calabi–Yau threefolds over $\mathbf{Q}$ for which the modularity has been established. As for non-rigid Calabi–Yau threefolds over $\mathbf{Q}$, the Langlands Program predicts that there should be some automorphic forms attached to them. I will consider some examples of non-rigid Calabi–Yau threefolds for which the modularity can be proved in the affirmative.

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Date: December 30, 2003.

Noriko Yui was partially supported by a Research Grant from NSERC, Canada.